Stability Diagram for Lift-Off and Blowout of a Round Jet Laminar Diffusion Flame

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An idealized model of a lifted flame above a round laminar jet is considered where diffusion rates of species and temperature are assumed equal but differential diffusion with respect to jet momentum is allowed. The combustion is described in terms of a global Arrhenius chemistry that is symmetric in fuel and oxidizer. Theoretical results on the propagation speeds of triple flames, and, the Landau–Squire solution for a nonreacting laminar round jet are combined to arrive at a transcendental equation for the lift-off height. For given chemistry, the stability behavior is controlled by a single Schmidt number, S, characterizing the differential diffusion between species (or temperature) and momentum and a parameter B, which is inversely proportional to the square root of the jet Reynolds number. Lift-off and blowout are characterized by a pair of critical curves in this two-dimensional parameter space the region between which corresponds to a stable lifted flame. A critical value of the Schmidt number exists above which the lift-off height increases continuously from zero on increasing the jet speed but below which the flame lifts off in a discontinuous manner through a subcritical bifurcation.

INTRODUCTION

The accurate control of flame stabilization is a key issue in many combustion processes, where recirculation of hot products and partial premixing are usually combined to ensure flame attachment. Flame stabilization is a complex process in which partial premixing and the physics of ignition and extinction are very important. To understand the basic mechanism at a fundamental level, the diffusion flame formed by a fuel jet in an oxidizing atmosphere has been studied extensively and stands as a useful model system for studying stabilization in a geometry that is relatively simple.

It is observed in a round jet laminar flame, that if the mass flow rate exceeds a critical value, the base of the diffusion flame lifts off from the burner tip and remains suspended at a certain distance above the burner. The phenomenon is known as "lift-off" and has been observed only in certain fuels and not in others. In theory, a further increase in the mass flow rate of the jet causes the lift-off height to increase until the base of the diffusion flame approaches the flame tip at which point the flame blows out. In most practical situations, the flow transitions to turbulence before lift-off is achieved or soon thereafter. However, under carefully controlled laboratory conditions, it is possible to observe the lift-off and blow-out behavior in the laminar regime [1–3] and this paper is exclusively devoted to the laminar lifted flame.

Lift-off is a considerably more difficult problem than blowout as the detailed structure of the viscous boundary layer near the tip of the nozzle and heat loss to the nozzle play dominant roles. Hysteresis effects are seen, that is, flame lift-off and reattachment do not happen at the same value of the mass flow rate. Near the critical conditions a complex time-dependent behavior where the flame jumps back and forth between the lifted and attached configurations can sometimes be observed. In this paper, we consider a flame that is already lifted and we assume that the mass flow rate is high enough so that the flame is stabilized in the far field region of the jet where the self-similar solution for a nonreacting round jet applies. Therefore, conditions very close to lift-off are not considered in detail.

The stabilization of a lifted flame in the far field of the jet is believed to be accomplished through the mechanism of the triple flame. A triple flame is a characteristic flame structure that has been observed experimentally as well as in numerical simulations wherever combustion occurs in a partially premixed regime. The flame

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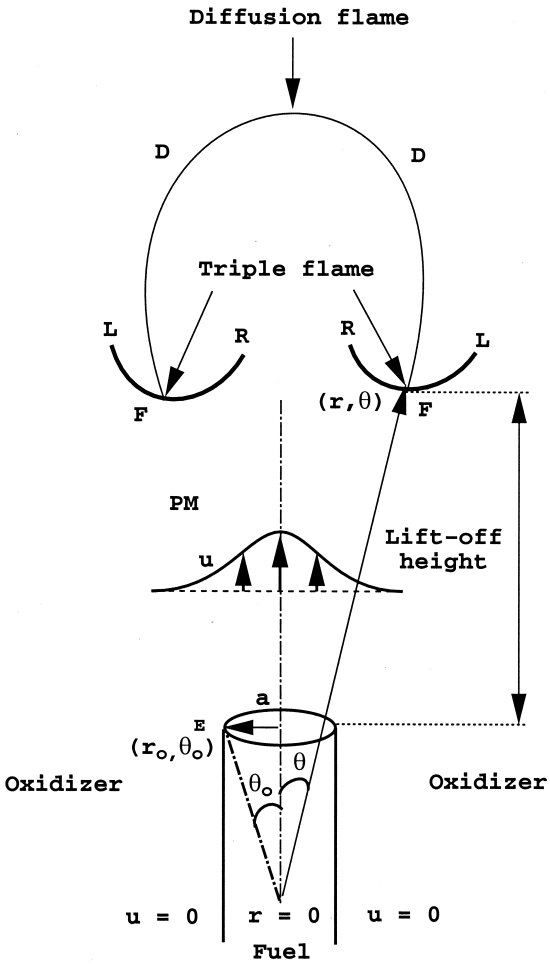

Fig. 1. Schematic of the coordinate system and sketch showing the location of the triple flame in a laminar lifted diffusion flame over a round jet.

stabilization mechanism is shown schematically in Fig. 1 in cross section. The fuel and oxidizer undergo partial premixing in the zone 'PM'. The mixture then burns in two branches, the fuel rich branch, 'FR' and the fuel lean branch 'FL'. Behind these two flame zones the hot streams of unburnt fuel and oxidizer come together and burn as a trailing diffusion flame, 'DF' along the stoichiometric surface. The structure consisting of the three branches FR, FL and DF is collectively known as the triple flame. The triple flame has a characteristic propagation speed determined by the local environment. The flame is stabilized where the flame propagation speed with respect to the fuel/oxidizer at rest matches the flow speed on the stoichiometric line.

The laminar lifted flame from a round jet has been studied in the context of the above triple flame picture by Chung and Lee [1] and Lee and Chung [2] using both theoretical and experimental methods. They approximated the triple flame speed as a constant equal to the stoichiometric planar flame speed and used the far field approximation to the Landau Squire solution for a jet issuing from a point source of momentum. The result of this analysis was a power law dependence of the lift-off height on the mass flow rate that was checked out reasonably well by their experiments. The analysis provided an interesting insight into the dominant role that the Schmidt number played in the lift-off phenomenon. For Schmidt number greater than unity or less than 0.5, the lift-off height increased with the mass flow rate as expected. However, for Schmidt numbers between 0.5 and 1, their liftoff formula showed the opposite behavior. They later showed by a linear stability analysis that the lifted flame was unstable for Schmidt numbers less than unity.

In this paper we attempt to improve upon the work of the previous authors by taking into account recent progress in understanding of the propagation properties of triple flames. The speed of a triple flame is reduced relative to the stoichiometric planar flame due to curvature effects and increased due to stream line deviation ahead of the triple flame on account of density changes due to heat release [4]. These effects are not necessarily small, in fact Lee and Chung report that in their experiment, the fluid velocity at the stabilization point was in fact measured to be larger than the stoichiometric flame speed, as would be expected for a triple flame dominated by heat release effects. Triple flame propagation speeds of about twice the stoichiometric speed have been observed.

We will attempt to formulate a simplified model problem that is analytically tractable and yet contains the essential physical features of a real jet flame. Thus, we consider a laminar round jet of fuel issuing from a thin nozzle into an oxidizing atmosphere. We will assume complete symmetry between fuel and oxidizer, that is, they have the same molecular weights and diffusivities and the reaction rate is invariant with respect to an interchange of fuel and oxidizer. This is of course a mathematical ide-

alization suitable for analytical study but is never strictly realized in the laboratory. Nevertheless, it serves as a useful baseline problem, an understanding of which is an essential first step. For the chemistry we will assume a global Arrhenius model where the fuel and oxidizer react directly to form the product through a reaction rate with a constant (temperature independent) pre-exponential factor [18]. Therefore, for given chemistry, the system is parameterized by three dimensionless numbers, the Reynolds number $\Re$ of the jet, the Schmidt number $S = \nu/k$ (ν is the kinematic viscosity and k is the mass diffusivity of fuel or oxidizer), and, the Lewis number $L = k_T/k$ (k_T is the constant thermal diffusivity of the gas). For a fixed jet Reynolds number, varying the Schmidt number changes the location of the Stoichiometric line in the flow field of the jet and this affects the lift-off height. Changing the Lewis number also affects the lift-off height through variation in the triple flame propagation speed. In this study we will consider the Lewis number to be fixed at unity and only consider the effect of varying the Schmidt number. Our results could in principle be generalized by incorporating Lewis number effects on triple flame speeds into the theoretical framework. The model problem thus formulated will be studied in the limit where the Reynolds number is large (and yet below the critical Reynolds number for transition to turbulence), the Zeldovich number characterizing the activation energy is large and the heat release parameter α (the relative rise in temperature along the stoichiometric line) is much less than unity. The first two conditions are well approximated in controlled laboratory experiments but the last is usually not since $\alpha \sim 0.8$ for undiluted hydrocarbon flames. However, the theoretical results based on the $\alpha \ll 1$ assumption has been found to agree quite well with full numerical simulations even in the case $\alpha \sim 0.8$ in studies with isolated triple flames [14].

In the following section we revisit the Landau–Squire solution for a round laminar jet and introduce some notation and terminology that are different but equivalent to that found in [1] and [2] as they are better suited to our analysis. Some recent results on triple flame theory are also summarized. In the subsequent section, the results of the previous two sections are combined to obtain a transcendental equation that gives the lift-off height as a function of the jet Reynolds number. We then use this equation for the lift-off height to analyze the lift-off/blowout characteristics of the different regions of parameter space. The conditions of validity of our simplified model are discussed before summarizing the principal results in the concluding section.

THE LANDAU–SQUIRE SOLUTION FOR THE ROUND JET

We consider an incompressible jet issuing through a nozzle of radius "a" into a stagnant atmosphere of the same fluid whose spatial extent is infinite. At distances large compared to the nozzle diameter (the far field) the self-similar solution corresponding to a point source of momentum [5–7] holds. In this paper, we will only use the approximate form of this solution valid for relatively large Reynolds numbers [6]

$$\psi_*(r_*, \theta) = \nu r_* f(\theta) \tag{1}$$

with $f(\theta)$ defined by

$$f(\theta) = \begin{cases} 4\theta^2/(\theta^2 + \theta_0^2) & \text{for } \theta \le \theta_0 \\ 2(1 + \cos\theta) & \text{otherwise.} \end{cases} \tag{2}$$

Here $\psi_*(r_*, \theta)$ is the stream function in polar co-ordinates (r_*, θ), ν is the kinematic viscosity and θ_0 is related to the jet Reynolds number

$$\Re \equiv \left(\frac{F_*}{2\pi\rho_*\nu^2}\right)^{1/2} \tag{3}$$

as

$$\Re = \frac{32}{3}\,\theta_0^{-2}. \tag{4}$$

F_* is the momentum injected by the jet in unit time, ρ_* is the density and the suffix '$*$' indicates that the quantity is in physical units. We are interested in the regime $\Re_{cr} > \Re \gg 1$ where $\Re_{cr}$ is the critical Reynolds number for transition to turbulence. This corresponds to a slender jet dominated by inertial rather than viscous effects, but the Reynolds number is still small enough that the jet does not transition to tur-

bulence. Such conditions can be achieved in controlled laboratory experiments (e.g., those of Chung and Lee [1]). Thus, θ_0 is a small dimensionless parameter characterizing the slenderness of the jet. We will treat θ_0 rather than the Reynolds number as the primary dimensionless quantity of interest.

The theory can be simplified by a choice of appropriate dimensionless variables. As a characteristic length scale, one can either choose 'a' the jet radius, or '$\ell_F \equiv k/U_s$' where ℓ_F is the flame thickness defined in terms of the thermal diffusivity 'k' and the laminar stoichiometric flame speed, 'U_s'. Of these, we choose the latter, as in the far field, the jet diameter is irrelevant. The appropriate scale factor for time is then ℓ_F/U_s. In terms of these dimensionless variables, the solution may be written as

$$u = \frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta} \tag{5}$$

$$v = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \tag{6}$$

$$\psi(r, \theta) = \psi_*/(U_s\ell_F^2) = Srf(\theta) \tag{7}$$

where (u, v) are the dimensionless velocity components in polar co-ordinates (r, θ) and $S \equiv \nu/k$ is the Schmidt number.

We may introduce the fuel mixture fraction 'Z', it obeys the equation for a passive scalar and has the value $Z = 1$ in pure fuel and $Z = 0$ in pure oxidizer. In the context of the Landau–Squire solution, the mixture fraction field Z can be written as [5]

$$Z = \frac{C}{r}\left(\frac{\theta_0^2}{\theta^2 + \theta_0^2}\right)^{2S} \tag{8}$$

where C is a constant. The solution has a singularity at the origin indicating that the solution breaks down in the near field. The singularity can be avoided by introducing a virtual origin at a distance 'r_0' below the jet exit and taking this point as the origin of the polar co-ordinate system. In order for the "jet thickness" to coincide with the diameter of the orifice at the jet exit, we must have $r_0 = a/\theta_0$. If we impose the condition that $Z = 1$ at $r_* = r_0$ and $\theta = 0$, we get $C = a/(\ell_F\theta_0)$. Thus, taking into account the virtual origin, Eq. (8) reduces to

$$Z = \frac{1}{\theta_0\ell r}\left(\frac{\theta_0^2}{\theta^2 + \theta_0^2}\right)^{2S} \tag{9}$$

where $\ell = \ell_F/a = k/(U_s a)$ is a dimensionless parameter which is usually very small.

RESULTS FROM TRIPLE FLAME THEORY

Triple flames were first identified and observed in a laboratory setting by Phillips [10]. Since then, this combustion regime has been the subject of many experimental studies [16, 17]. They were also seen in a variety of numerical simulations (see Vervisch and Poinsot [11] for a review). Dold [12] and Hartley and Dold [13] proposed the first theoretical analysis of triple flames based on the method of activation energy asymptotics. The analysis of Dold assumed that the upstream gradient of mixture fraction is small, implying a weakly curved triple flame, and further, neglected any effect of density changes due to heat release. The subsequent paper of Hartley and Dold [13] relaxed the first assumption and presented a theory where the speed of propagation of the triple flame is determined as the solution of an integral equation for a moderately curved flame. For a weakly curved flame, this relation reduces to the algebraic equation due to Dold for determining the flame speed. Heat release in a flame may be characterized by the heat release parameter $\alpha = (T_s - T_0)/T_s$, where T_0 is the temperature of the fresh gases and T_s is the adiabatic temperature of a stoichiometric flame. Clearly, $1 > \alpha > 0$. In typical hydrocarbon flames, $\alpha \sim 0.8$, neglecting heat release effects correspond to the limit $\alpha \to 0$. The effect of heat release on triple flames was studied through numerical simulation of the compressible fluid equations with a global Arrhenius reaction between fuel and oxidizer by Ruetsch et al. [4]. They showed that heat release effects play an important, often the dominant role in triple flame propagation. The expansion caused by heat release behind the flame front results in a deviation of the streamlines in front of the flame, in turn resulting in an increase in triple flame speed. This increase can

in fact off-set the reduction in speed due to flame front curvature, so that, a weakly curved triple-flame actually can propagate faster than a planar flame.

A triple flame solution [14], based on an approximation of the curved flame front by a parabolic profile, has recently been developed in the limit of large activation energy and small but finite heat release. A closed form expression for the triple-flame propagation speed is obtained that accounts for the combined effect of heat release and flame front curvature. Comparison to numerical simulation of the primitive equations shows an acceptable agreement, even for $\alpha \sim 0.8$. In the case of a weakly curved flame, this triple flame solution is most transparent and the easiest to interpret. This is the one that we will use to determine the lift-off height. These results, obtained on the assumptions $\mu\beta \ll 1$ and $\alpha \ll 1$ are

$$U = 1 + \alpha - \kappa, \tag{10}$$

$$\kappa = \frac{\beta\mu(\alpha)}{\sqrt{4n - 2}}, \tag{11}$$

$$\mu(\alpha) = \frac{\mu(0)}{1 + \alpha}. \tag{12}$$

Here U is the triple flame propagation speed normalized by the stoichiometric planar flame speed, κ is the curvature of the flame tip measured in flame units ℓ_F, $\mu(\alpha)$ is a normalized mixture fraction gradient $\ell_F Z_s^{-1}(dZ/dy)_s$ at the triple flame location, and $\mu(0)$ is the corresponding mixture fraction gradient in the absence of the flame (or equivalently, the mixture fraction gradient for a flame with negligible heat release). The justification for using the reduced forms corresponding to $\mu\beta \ll 1$ is postponed to a later section. The fluid is assumed to consist of pure fuel (mass fraction Y_F) and pure oxidizer (mass fraction Y_O) reacting through an Arrhenius reaction rate $BY_F^n Y_O^n \exp(-T/T_a)$. The Zeldovich number $\beta \equiv (T_a/T_s)\alpha$ is a measure of the temperature sensitivity of the reactions. All transport coefficients are assumed constant and the molecular weights and diffusivities of fuel and oxidizer are assumed equal. It is seen that the presence of heat release affects the flame in two ways:

1. The triple flame speed is increased by the additive term α which accounts for the lowering of the flow speed immediately ahead of the flame by the combined effect of flow divergence due to thermal expansion and mass conservation.
2. The mixture fraction gradient is reduced due to the pulling apart of adjacent streamlines which in turn causes the flame curvature to be less, accordingly the flame propagates faster because heat losses are reduced.

For very weakly curved hydrocarbon flames, $\alpha \approx 0.8$ and $\kappa \approx 0$, so that $U \approx 1.8$, consistent with the experimental observation [15] that the flow speed at the base of the lifted flame (equal to the triple flame speed) is almost twice the adiabatic flame speed for the corresponding stoichiometric mixture.

LIFT-OFF HEIGHT

We consider now the case of a stream of pure fuel flowing into an initially stagnant oxidizing atmosphere of infinite extent. If the mixture is lighted, a steady diffusion flame would be established after a period of time sufficiently long for the initial disturbances to decay. The location of the diffusion flame is then coincident with the stoichiometric surface $Z = Z_s = 1/2$, or, using Eq. (9),

$$r = \frac{2}{\ell\theta_0}\left(\frac{\theta_0^2}{\theta^2 + \theta_0^2}\right)^{2S} \tag{13}$$

gives the shape of the diffusion flame. The dimensionless mixture fraction gradient

$$\mu = \left|\frac{2}{r}\frac{\partial Z}{\partial\theta}\right| \tag{14}$$

may be obtained by differentiating (9) and eliminating the variable r using (13),

$$\mu = 2S\ell\,\frac{\theta}{\theta_0}\left(1 + \frac{\theta^2}{\theta_0^2}\right)^{2S-1}. \tag{15}$$

Because we are only concerned with the far field, the radial component of the mixture fraction gradient is negligible, so that, the tangential component given by Eq. (15) can be considered equal to the component of this vector perpendicular to the stoichiometric line. Similarly, the

flow speed at the stoichiometric line in a direction parallel to it may be determined from Eq. 1 as follows:

$$u = \frac{4S\ell}{\theta_0}\left(1 + \frac{\theta^2}{\theta_0^2}\right)^{2S-2} \tag{16}$$

The lift-off "height", θ is determined by equating the flow speed at the triple flame location Eq. (16) with the triple flame speed determined from Eq. (10) with (15) as the mixture fraction gradient $\mu(0)$. After some simplification, we obtain the following transcendental equation for determining the normalized lift-off height $x = \theta/\theta_0$:

$$f(x) \equiv (1 + x^2)^{2S-2}[1 + Ax(1 + x^2)] = B \tag{17}$$

where

$$A = \frac{\beta\theta_0}{2(1 + \alpha)\sqrt{4n - 2}} \tag{18}$$

$$B = \frac{\theta_0}{4S\ell}(1 + \alpha) \tag{19}$$

In Eq. (16) ℓ and θ_0 are both very small, however, for a lifted flame to exist we must have $u \sim 1$, so that $\ell \sim \theta_0$. Therefore, $A \sim \mu\beta \ll 1$, since we have assumed a weakly curved triple flame, but, $B \sim O(1)$. The inverse of the parameter B is proportional to the square root of the jet Reynolds number, $\Re$ and in this paper we will use B^{-1} rather than $\Re$ to characterize the strength of the jet. This allows a simpler description free from cumbersome numerical scale factors. Similarly, we will describe the lift-off height using the variable $x = \theta/\theta_0$ instead of calculating the actual height above the nozzle ($r \cos\theta - r_0 \cos\theta_0$) which may easily be obtained from x using simple co-ordinate transformations. The "n" in (18) refers to the composition exponent in the Arrhenius rate expression defined earlier.

ANALYSIS OF THE LIFT-OFF HEIGHT EQUATION

The nature of solutions of the transcendental Eq. (17) depends on the Schmidt number. We will consider the two cases $S \geq 1$ and $S < 1$ separately.

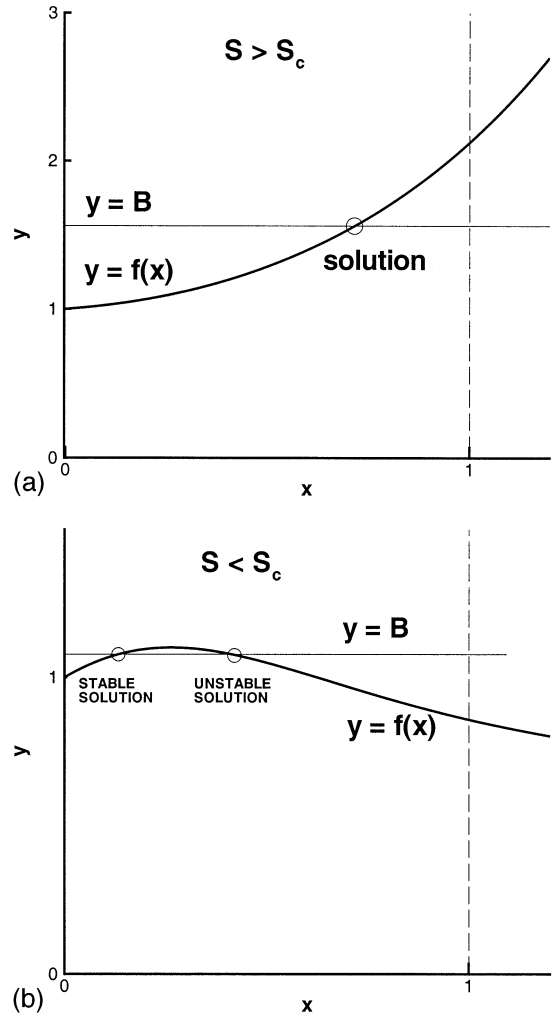

Fig. 2. Sketch showing the solutions of the transcendental equation determining the lift-off height (A) $S > S_c$ (B) $S < S_c$.

Schmidt Number Not Less Than Unity

Physical solutions of Eq. (17) must be in the range $1 \geq x \geq 0$, with the limits $x = 0$ and $x = 1$ corresponding to a flame at blowout and an attached flame, respectively. Figure 2a is a sketch of the function $f(x)$, which is monotonically increasing because $A \geq 0$ and $S \geq 1$. Clearly, a solution for x exists, provided $1 \leq B \leq B_m$ where

$$B_m = f(1) = 4^{S-1}(1 + 2A). \tag{20}$$

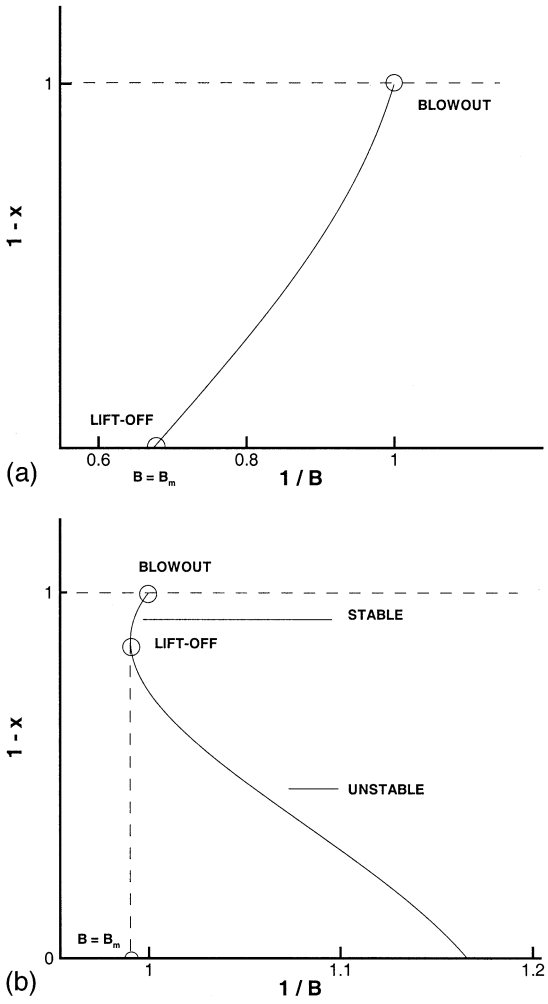

Fig. 3. The bifurcation diagram of $1 - x$ vs. $1/B$ showing lift-off and blowout when (a) $S > S_c$ (b) $S < S_c$.

The solution is unique. At $B = B_m$, $x = 1$, this corresponds to "lift-off". For $B > B_m$ the flame remains attached. At $B = 1$, $x = 0$, and this corresponds to "blow-out". No flame is possible if $B < 1$. A sketch of $1 - x$, a dimensionless measure of the lift-off height with B^{-1} is shown in Fig. 3a. The bifurcation from an attached to a lifted flame is seen to be supercritical.

At the intersection of B and $f(x)$, $f'(x) > 0$. If we trace back the origin of the various terms in Eq. (17), the following interpretation for the positive slope is clear. The local flow speed and the local triple flame speed can in general either increase or decrease with x, depending on the value of the Schmidt number, S. A positive slope at the solution "x" indicates that on a slight displacement upward from the equilibrium position, the increase in triple flame speed is *more* than any increase in the oncoming flow speed. Conversely, for a slight displacement downward from the equilibrium position the decrease in triple flame speed is *more* than the decrease in the fluid speed at the flame location. In other words, the flame position is stable to such displacements. On the other hand, if $f'(x) < 0$, the flame would have been unstable. A slight upward or downward displacement would cause it to move away from the equilibrium location.

The above argument does not of course prove stability against all small perturbations. Neither does it prove that the attached flame is unstable for $B < B_m$. However, we will accept these as experimental facts.

Schmidt Number Less Than Unity

In this case, Eq. (17) allows for more complex behavior. In the case $A = 0$, $f(x)$ decreases monotonically with x, because $S < 1$. Therefore, the only solution is unstable, since $f'(x) < 0$. Setting $A = 0$ is equivalent to neglecting curvature effects in the triple flame propagation speed. Under this assumption no stable solutions are possible for $S < 1$ as pointed out by Lee & Chung [2]. However, the situation changes when one considers small ($0 < A \ll 1$) nonzero values of A. It is clear from Eq. (17) that for $x \to 0$, $f(x) \sim 1 + Ax$, increases with x. When x is not close to zero, the dominant term is $(1 + x^2)^{2S-2}$, and this decreases with x. Therefore a maximum of the function $f(x)$ exists in the interval $0 \leq x \leq 1$ if A is sufficiently small.

Let us denote this maximum point by $x = x_m$ and assume $x_m < 1$. Let $B_m = f(x_m)$. If $B > B_m$ no solutions exist (Fig. 2b). If B is slightly less than B_m, a pair of solutions exist corresponding to $f'(x) > 0$ and $f'(x) < 0$, respectively. The former is stable, the latter is unstable. Figure 3b shows the bifurcation diagram. The lift-off height defined as $1 - x$ is plotted against B^{-1}, a measure of the jet momentum. The bifurcation to the lifted flame is seen to be subcritical. Of the two branches in Fig. 3b, the lower one corresponds to the unstable solution and the upper branch corresponds to the stable solution. In this situation lift-off is seen to be

discontinuous. Upon being subjected to a large enough perturbation the flame may suddenly lift-off to a finite height above the burner.

A time dependent bi-stable state where the flame rapidly jumps between the lifted and attached configurations could happen in the neighborhood of the point $B = B_m$. Such behavior near to lift-off has indeed been observed. A detailed consideration of such behavior will require considerations of heat loss to the burner lip. Another consequence of the subcritical nature of the bifurcation is that, the lift-off may not happen at the same mass flow rate for every realization of the experiment. The reason is, the transition to a lifted flame is possible before the linear stability limit is reached but it requires a finite perturbation to accomplish. Such external perturbations depend on ever present fluctuations of a random and irreproducible nature.

The actual location of the maximum x_m can be found from the condition $f'(x_m) = 0$. This gives a quartic equation for x_m. It is convenient to rearrange this equation and write it as

$$x_m = \frac{A(1 + x_m^2)(1 + 3x_m^2)}{4(1 - S)[1 + Ax_m(1 + x_m^2)]}. \tag{21}$$

This can be solved iteratively. Because $x_m = 0$ when $A = 0$, $x_m = 0$ can be taken as the zeroth iteration. Substituting $x_m = 0$ on the right hand side we obtain the leading term in the expansion of x_m in powers of the small parameter A:

$$x_m = \frac{A}{4(1 - S)} + \cdots \tag{22}$$

and hence

$$B_m = f(x_m) = 1 + \frac{A^2}{8(1 - S)} + \cdots. \tag{23}$$

The singularity at $S = 1$ is a consequence of the fact that as $S \to 1$, from (17), $x_m \to \infty$.

We have seen that the bifurcation to a lifted flame is always supercritical when $S > 1$ but can be subcritical in the $S < 1$ case. The critical Schmidt number $S_c (S_c < 1)$ for this boundary between supercritical and subcritical behavior is achieved when the maximum of the function $f(x)$, $x = x_m$ becomes a point of inflexion.

For Schmidt numbers larger than S_c the function $f(x)$ is monotonic in the range $0 \le x \le 1$, so there is only one stable solution corresponding to a lifted flame provided $1 \ge B \ge B_m$. The bifurcation from the attached to the lifted state is therefore supercritical. For Schmidt numbers smaller than S_c, there are two solutions corresponding to a lifted flame and the bifurcation is subcritical. When $S > S_c$, the condition of lift-off is $B \equiv B_m = f(1)$ so that the lift-off boundary in $S - B$ space is given by Eq. (20). For $S < S_c$ it is given by (23). The blowout condition in all cases correspond to $B = 1$ because this is the point at which the stable branch of the solution approaches the flame tip $x = 0$. For $B < 1$ no stable solutions exist.

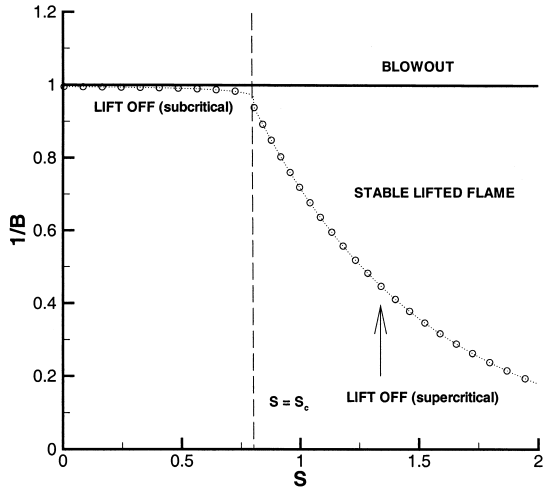

Fig. 4. The parameter space $S - B^{-1}$ showing the zone of lifted flame solutions and the lift-off/blowout limits for A = 0.2.

All of this information is condensed in the stability diagram of Fig. 4 showing the critical curves for lift-off and blowout. A stable lifted flame can only be supported in the region between the two curves. For $S < S_c$, the bifurcation to a lifted flame is subcritical but for $S > S_c$ it is supercritical. The dashed line is obtained if (21) is solved numerically and the symbols correspond to (23) for $S < S_c$. For $S > S_c$ both dashed line and symbols correspond to Eq. (20).

DISCUSSION

In this analysis, the Landau–Squire solution for the round laminar jet was combined with theo-

retical results for triple flame propagation, to gain some understanding of the laminar lifted flame problem. However, in addition to the assumption of the Lewis numbers being equal for all species, we borrowed theoretical results from triple flame theory that apply strictly to planar flames. In the case of the lifted flame from a round jet, the flame is axially symmetric instead of planar, and further, the jet velocity is not quite uniform in space. For the theory to be applicable, the radius of curvature of the triple-flame needs to be much smaller than the radius of the flame cone. Further, the variation of the jet velocity over a distance of the order of the radius of curvature of the triple flame should be small. We now examine the conditions under which these assumptions are true.

By using Eqs. 5, 11, 12, and 15 we have

$$\kappa\left[\frac{1}{ur}\frac{\partial u}{\partial \theta}\right]^{-1} = \frac{\beta S}{(1+\alpha)\sqrt{4n-2}} \tag{24}$$

Any effect of the nonuniformity of the jet velocity field at the stabilization point can be neglected if the quantity on the right is much greater than unity. This condition is certainly true in the asymptotic limit $\alpha \to 0$ and $\beta \to \infty$. For a typical hydrocarbon flame of pure fuel burning in air, $\alpha \approx 0.8$, $n \approx 1$, $\beta \approx 8$, and $S \sim 1$, so that the right hand side of Eq. 24 is approximately 3.2.

A comparison of the relative magnitudes of triple flame curvature and the curvature of the partially premixed front due to the axisymmetric geometry can also be made. The radius of curvature of the flame ring at the base of the lifted flame is $R \approx r\theta$. By using Eqs. (11), (12), and (13) we easily derive the following relation between the two principal curvatures of the diffusion flame edge.

$$\kappa R = \frac{2\beta S}{(1+\alpha)\sqrt{4n-2}}\left[1+\frac{\theta_0^2}{\theta^2}\right]^{-1} \tag{25}$$

If $\theta/\theta_0 \sim 1$, the right hand side is very large in the limit of large activation energy, $\beta \to \infty$ so that using the solution for a planar flame is justified. However, at the flame tip, the radius of curvature of the base of the flame cone goes to zero, so that the assumption of a planar triple flame must break down. If $\kappa R \sim 1$ is considered the limit at which the assumption of a planar flame breaks down, then

$$\frac{\theta_0^2}{\theta^2} < \frac{2\beta S}{(1+\alpha)\sqrt{4n-2}} - 1 \tag{26}$$

defines the limit of validity of the approximation. In the limit of $\beta \to \infty$, the planar solution for the triple flame is applicable except for an infinitely small region near the flame tip. However, for the typical numerical values for hydrocarbon flames used in the last paragraph, we have $\theta/\theta_0 > 0.4$. Therefore, though neglect of the curvature of the flame cone is consistent with the large Zeldovich number assumption, corrections due to this curvature could be important for real hydrocarbon flames.

In using the theoretical expression for the triple flame speed, we have used Eqs. (11) and (12) that correspond to a weakly curved triple flame. This means that β multiplied by the mixture fraction gradient in flame units must be small, that is,

$$\frac{2\beta S\ell}{(1+\alpha)\sqrt{4n-2}}\left(\frac{\theta}{\theta_0}\right)\left[1+\frac{\theta^2}{\theta_0^2}\right]^{2S-1} \ll 1 \tag{27}$$

This condition is satisfied provided $\ell = \ell_F/a \ll \beta^{-1}$, that is, if the flame itself is much thinner than the nozzle diameter. Because $\ell \sim \theta_0$, this condition is also equivalent to $A \ll 1$.

CONCLUSION

In an earlier work, Chung and Lee have shown that the lift-off height for a laminar lifted flame may be calculated based on a balance between the triple flame propagation speed and the jet velocity assuming that the triple flame speed is approximately equal to the planar flame speed. In this paper we have extended and refined this previous work by using analytical results for the triple flame speed that account for deviations of the triple flame speed from the planar flame speed due to effects of heat release and curvature. We find that stable lifted flames are possible for all values of the Schmidt number, however, the region of parameter space that supports a lifted laminar flame is very narrow for Schmidt numbers less than a critical value $S_c \sim 1$. This is a modification of the earlier

result of Chung and Lee who found that flames with Schmidt numbers less than unity must be unstable. The present analysis reveals an interesting feature of the transition to a lifted flame. A curve is identified in the parameter space defined by the Schmidt number and jet Reynolds number that separates the region where the transition to lift-off is subcritical from the zone where it is supercritical. In the subcritical case, the flame can lift-off to a finite distance above the nozzle without necessarily going through the intermediate locations. Further, the transition point can depend on external perturbations and may not be exactly reproducible. Oscillatory behavior between the two simultaneously existing linearly stable solutions is possible. This might explain some of the qualitative effects seen in a laminar jet flame close to lift-off.

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