A numerical study of self-similarity in a turbulent plane wake using large-eddy simulation

Sandip Ghosal^{a)} Center for Turbulence Research, Stanford University, Stanford, California 94305

Michael M. Rogers NASA-Ames Research Center, Moffett Field, California 94035

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Turbulent wakes are known to develop self-similarly sufficiently far downstream from obstacles that generate them. It has long been assumed that the spreading rate of the wake in the self-similar regime is independent of the details of the body generating the wake, being dependent only on the total drag (or momentum deficit). This assumption seems to be in contradiction with some recent experiments. In this study we attempt to complement these experimental investigations through a numerical study of a time-developing wake. A numerical study has the advantage of eliminating many of the uncontrolled factors present in experiments and allowing precise control of initial conditions. Large-eddy simulations employing the recently developed dynamic localization model are used to extend previous results from direct numerical simulations. The large-eddy simulation results are compared to the direct numerical simulation database, wherever such comparisons are feasible, as a check of the method. Like the experiments, the large-eddy simulations suggest that non-unique self-similar states, characterized by different spreading rates and turbulent statistics, are possible and that they can be maintained for significant time periods. The study also demonstrates the predictive capability of the dynamic localization subgrid model. © *1997 American Institute of Physics.* [S1070-6631(97)02006-0]

I. INTRODUCTION

A turbulent flow is said to be self-similar when some or all of its statistical properties depend only on certain combinations of the independent variables rather than on each independent variable individually. The consequence of this is that the number of independent variables in the problem is reduced, thus greatly facilitating its solution. Geometrically, a self-similar flow possesses a certain symmetry; for example the flow pattern on any two cross sections perpendicular to a given axis may be identical except for a scale factor. The property of self-similarity has been used on many occasions in fluid dynamics to derive elegant solutions to otherwise very difficult problems (such as the structure of turbulent boundary layers, jets and wakes). Recently, George¹ presented a critical analysis of the self-similarity argument in the context of certain apparent discrepancies of self-similar solutions with experimental results on jets and wakes. He argued that in the traditional analysis, in addition to the assumption of self-similarity, one often invokes additional restrictions inspired by the dictum "turbulence forgets its initial conditions." For example, in the case of the turbulent plane wake one requires that the growth rate sufficiently far from the source can depend only on the momentum deficit of the wake (which is proportional to the drag on the obstacle producing the wake). Dimensional analysis then implies "universal" solutions that do not depend on the nature of the obstacle or the details of the initial conditions. George argued that when such additional restrictions are removed, one

obtains a wider class of self-similar solutions that are no longer "universal." Thus, for a plane wake, these solutions will depend on the nature of the obstacle and not just on the total drag. These conclusions seem to be in agreement with the experiments of Wygnanski *et al.*² and Marasli *et al.*³ However, conditions in experiments are difficult to control precisely and some doubt remains about whether the results indicate the existence of multiple self-similar states or if this is an artifact of experimental uncertainties.

To address this issue further, direct numerical simulations (DNS) of plane wakes have been generated by Moser and Rogers⁴ and Moser, Rogers, and Ewing.⁵ Such numerical simulations are free from various uncontrollable extraneous factors that complicate the interpretation of experiments and should complement the experimental results already available (see Refs. 2 and 3, and references therein). However, such simulations are very costly since all scales of turbulent motion must be accurately resolved. In practice this limits the Reynolds numbers and the extent of flow evolution that can be simulated. This suggests that large-eddy simulation (LES) might be a better tool than direct numerical simulation to study high-Reynolds-number fully developed wake turbulence over long evolution times, particularly if smallscale information is not desired. In LES one explicitly solves a coarse-grained version of the Navier-Stokes equations. The collective effect of the small scales on the large scales is taken into account through a "subgrid model." Although LES can be computationally much less expensive, it has the disadvantage that it leaves open the possibility of significant errors resulting from the approximation of the unknown subgrid stress by a model.

^{a)}Present address: LMFN-INSA Rouen, URA-CNRS 230-CORIA, 76821 Mont-St-Aignan, France.

In order to achieve the longest possible simulation of a high-Reynolds-number self-similar plane wake we have resorted to LES of the temporally evolving flow, as was done in the DNS. First, the LES methodology is validated by comparison to existing DNS for cases that are less computationally intensive. The limitations of the DNS can then in turn be addressed by the use of new LES results in larger computational domains. This complementary use of both LES and DNS bolsters confidence in the simulation results and facilitates better understanding of the self-similar behavior of the plane wake.

Three DNS of temporally evolving plane wakes have been documented in Moser, Rogers, and Ewing.⁵ These wakes differ from each other in the level of initial twodimensional turbulent fluctuations. The "unforced" case is initiated from two realizations of a fully developed turbulent boundary layer with no added disturbances. In the other "weakly forced" and "strongly forced" cases, additional two-dimensional fluctuation energy has been added to the boundary layer turbulence. This is achieved by multiplying the streamwise and cross-stream velocity components associated with the two-dimensional Fourier modes in the computation by factors of 5 and 20, respectively. The resulting evolution of the unforced and weakly forced cases shows convincing evidence of self-similar evolution, although the growth rates and Reynolds stress levels for the two cases are different. The strongly forced case, on the other hand, shows irregularities in the shapes of mean velocity and Reynolds stress profiles and exhibits at most a short period of approximate self-similar evolution (with a very high growth rate and large levels of Reynolds stress).

The flow structure in this strongly forced case has an underlying pattern of a few large-scale motions and it was speculated in Ref. 5 that the poor self-similarity resulted from an inadequate sample of large-scale turbulent eddies in the computational domain. In order to confirm this, a LES of nominally the same flow in a domain that is twice as large in the streamwise direction and four times as large in the spanwise direction has been generated and compared to both DNS and LES of the small-domain case. Of primary interest is whether a self-similar state does indeed exist in this strongly forced flow and what the growth rate and Reynolds stress levels are if such a period exists. This allows us to better address the issue of whether or not multiple initialcondition-dependent self-similar states exist for the turbulent plane wake.

The LES is performed using a fully spectral code and a recently developed subgrid model known as the "dynamic localization model" (DLM). In a previous paper,⁶ the theoretical development leading to the dynamic localization model for large-eddy simulation was presented. The method has been successfully applied to isotropic turbulence,^{6,7} channel flow,⁸ and the flow over a backward-facing step.^{9,10} Two attractive features of this model are:

(1) The magnitude of the eddy viscosity does not need to be prescribed in an *ad hoc* manner but the algorithm itself chooses an optimum value based on a certain welldefined optimization procedure. (2) The subgrid model parameter ("Smagorinsky coefficient") is a function of space and time and automatically adjusts itself to the intensity of the turbulence. In particular, it goes to zero near walls and vanishes in those regions of space and time where the flow is laminar.

The present study, in addition to investigating the issue of self-similarity in plane wakes, also provides another test of the predictive capability of the dynamic localization model. This includes both the ability to predict turbulent statistics as well as flow structure. In Moser, Rogers, and Ewing,⁵ forcing was found to significantly affect both statistics and flow structure in the plane wake, with forcing increasing the level of organized large-scale motions in the flow. Since it seems that these differences in flow structure are linked to the differences in turbulent statistics it may be essential that the subgrid model preserve the character of the filtered vorticity field for accurate prediction of the statistics. The local character of the subgrid model employed in this work would seem to offer a greater likelihood of achieving this. The level of correspondence between the vortex structures in the LES and the DNS is also of interest for flow control and understanding mechanisms of turbulent mixing.

In Sec. II certain general properties of plane wakes are reviewed and the problem to be solved numerically is defined. In Sec. III the computational methods used, including the subgrid model, are briefly discussed. The LES results for the unforced wake are presented and compared with the DNS database in Sec. IV. LES computations of the forced case in two different domain sizes (the smaller for comparison to the DNS, the larger to address limitations of the DNS and to study the long-time evolution of the forced case) are considered in Sec. V. In the concluding Sec. VI, the results and their significance are discussed.

II. FORMULATION OF THE PROBLEM

In a temporally developing wake the flow is statistically homogeneous in the streamwise (x) and spanwise (z) directions and inhomogeneous in the cross-stream (y) direction. The governing equations are the incompressible Navier– Stokes equations with periodic boundary conditions in x and z. In the y direction the domain is infinite and the velocity field is assumed to asymptotically approach the free-stream velocity, which can be taken as zero in a suitably chosen reference frame. This flow becomes equivalent to the physically more relevant spatially developing wake in the limit of a small wake deficit. If one imagines a "box" being advected downstream at the "free-stream" velocity in a spatially developing wake, then the motion of the fluid in the imaginary box approximates a temporally developing wake. The integrated mass flux deficit

$$\mu = -\int_{-\infty}^{+\infty} \delta U(y) dy \tag{1}$$

is conserved in a temporally developing wake, as opposed to the momentum flux deficit

$$\mu_* = -\int_{-\infty}^{+\infty} (U_\infty + \delta U(y)) \,\delta U(y) \,dy, \qquad (2)$$

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which is conserved for a spatially developing wake. Clearly, if the mean velocity deficit δU is small compared to the free stream velocity U_{∞} , then $\mu_* \approx U_{\infty}\mu$. A suitable scale for the velocity is the initial centerplane velocity deficit $\delta U_0 = -(\delta U(0))_{t=0}$ and a suitable length scale is $\mu/\delta U_0$. The associated time scale is $\mu/(\delta U_0)^2$. Most of the results given below are quoted in these units.

III. COMPUTATIONAL METHODS

The numerical method used is a spectral method in vorticity variables. Both the velocity and vorticity are periodic in the x and z directions and can therefore be expanded in a basis of trigonometric functions for these variables. The y direction is somewhat more difficult to deal with since the domain is infinite in y. One method is to choose a basis of functions that have an infinite support (such as the Jacobi polynomials coupled with a mapping to the infinite interval) for the y direction.¹¹ However, here we use an artifice that results in a simpler numerical code. We take advantage of the fact that in a wake the vorticity field is much more confined in the y direction than the velocity field. One then expands the vorticity in a trigonometric series in y defined over (y_{min}, y_{max}) with periodic boundary conditions. This is permissible provided that the vorticity is narrowly confined around y=0 and effectively decays to zero at the boundaries y_{min} and y_{max} . The velocity field is not so confined and cannot be represented in terms of these trigonometric functions. But once the vorticity field is determined, the correct velocity field may be obtained by adding a potential "correction" to the periodic velocity field so as to match the boundary conditions at $y = \pm \infty$. Further details of the computational method may be found in Corral and Jimenez.¹².

We use the "dynamic method" for computing the coefficient $C(\mathbf{x},t)$ in the generalization of Smagorinsky's subgrid model

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2C(\mathbf{x}, t) \Delta^2 |\overline{S}| \overline{S}_{ij}, \qquad (3)$$

where τ_{ij} is the subgrid stress, \overline{S}_{ij} is the resolved rate of strain, $|\overline{S}|^2 = 2\overline{S}_{ij}\overline{S}_{ij}$, and Δ is the LES filter-width (taken equal to the grid spacing). We will consider two variants of the dynamic method for determining *C*. The first, the Dynamic Model (DM), can be considered as a special case of the more general DLM discussed below for flows that are homogeneous in one or more directions. For the wake flow the coefficient *C* is considered a function of *y* and *t* only in DM and is given by

$$C(y,t) = \frac{\langle m_{ij} L_{ij} \rangle_{xz}}{\langle m_{kl} m_{kl} \rangle_{xz}},$$
(4)

where the angular brackets denote averaging over the homogeneous x-z planes. Here $L_{ij} = \widehat{u_i u_j} - \widehat{u_i} \widehat{u_j}$ is the Leonard term and $m_{ij} = \Delta^2 |\widehat{S|S_{ij}} - \widehat{\Delta}^2 |\widehat{S}| \widehat{S}_{ij}$, where $\overline{u_i}$ is the filtered velocity and the \wedge denotes the ''test filtering'' operation:

$$\hat{f}(\mathbf{x}) = \int G(\mathbf{x}, \mathbf{y}) f(\mathbf{y}) d\mathbf{y}.$$
(5)

The "test-level" filter-width is $\hat{\Delta}$ ($\hat{\Delta} > \hat{\Delta}$) and $G(\mathbf{x}, \mathbf{y})$ is the test filter kernel.

The second method, DLM, is applicable to arbitrary homogeneous flows but imposes the constraint $C \ge 0$. It is the more general of the two but requires more computation. The constraint $C \ge 0$ can be relaxed. This is done either by introducing an additional equation for the subgrid energy, k (Ref. 6) or by adding a "stochastic backscatter" term⁷ In DLM one obtains $C(\mathbf{x}, t)$ as a function of position at each time-step by solving an integral equation

$$C(\mathbf{x}) = \left[f(\mathbf{x}) + \int \mathcal{H}(\mathbf{x}, \mathbf{y}) C(\mathbf{y}) d\mathbf{y} \right]_{+}, \qquad (6)$$

where the suffix "+" indicates the positive part and

$$f(\mathbf{x}) = \frac{1}{\alpha_{kl}(\mathbf{x}) \alpha_{kl}(\mathbf{x})} \bigg[\alpha_{ij}(\mathbf{x}) L_{ij}(\mathbf{x}) - \beta_{ij}(\mathbf{x}) \int L_{ij}(\mathbf{y}) G(\mathbf{y}, \mathbf{x}) d\mathbf{y} \bigg],$$
(7)

$$\mathscr{K}(\mathbf{x},\mathbf{y}) = \frac{\mathscr{K}_A(\mathbf{x},\mathbf{y}) + \mathscr{K}_A(\mathbf{y},\mathbf{x}) - \mathscr{K}_S(\mathbf{x},\mathbf{y})}{\alpha_{kl}(\mathbf{x})\alpha_{kl}(\mathbf{x})},$$
(8)

$$\mathscr{H}_{A}(\mathbf{x},\mathbf{y}) = \alpha_{ij}(\mathbf{x})\beta_{ij}(\mathbf{y})G(\mathbf{x},\mathbf{y}), \qquad (9)$$

and

$$\mathscr{H}_{S}(\mathbf{x},\mathbf{y}) = \boldsymbol{\beta}_{ij}(\mathbf{x})\boldsymbol{\beta}_{ij}(\mathbf{y}) \int G(\mathbf{z},\mathbf{x})G(\mathbf{z},\mathbf{y})d\mathbf{z}.$$
 (10)

In these expressions $G(\mathbf{x}, \mathbf{y})$ is the "test filter," $\alpha_{ij} = -2\hat{\Delta}^2 |\hat{\overline{S}}|_{ij}, \beta_{ij} = -2\Delta^2 |\overline{S}|_{ij}$, and L_{ij} is the Leonard term. The method of numerically solving the integral equation to determine the coefficient *C* has been described elsewhere.⁶ The test filter-width in these computations was taken to be twice the grid-filter width, $\hat{\Delta}=2\Delta$, and a "tophat" filter was used with a Simpson's rule quadrature.

IV. VALIDATION OF THE LES FOR THE UNFORCED WAKE

In this section we attempt to establish confidence in the predictive capability of the subgrid model by reproducing, using LES, the results for the 'unforced' plane wake generated by DNS in Moser and Rogers⁴ and Moser, Rogers, and Ewing.⁵ The initial conditions for the DNS were generated by taking two realizations of 'turbulence over a flat plate' from DNS data generated by Spalart¹³ and 'fusing' them together to produce a wake. Physically this corresponds to a situation in which two independent boundary layers exist on either side of a rigid plate and the plate is instantaneously ''dissolved'' without disturbing the surrounding fluid. This initial DNS data field was then interpolated onto the coarser LES grid to generate the initial conditions for the LES. All the parameters in the LES described in this section were chosen to correspond to those used in the DNS.

The LES reported here were performed on a grid of size $N_x = 64$, $N_y = 48$, and $N_z = 16$. By contrast the DNS required up to $N_x = 512$, $N_y = 195$, and $N_z = 128$ modes. The half-size of the y-domain was set to $Y_0 = 16$. To compare the LES results to the DNS, all DNS data must first be "filtered" to

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FIG. 1. The square of the wake width as a function of time using DLM —; DM---; No-model \cdots ; filtered DNS \bullet .

the same resolution as the LES. This is done by truncating the DNS data in Fourier space to the same number of modes retained in the LES for the x and z directions. For the y direction, the DNS data are interpolated onto the coarser LES grid. This filtering procedure is applied to the initial conditions as well as to all DNS data with which we wish to compare the LES results. The "filtered DNS" represents the theoretical best that can be achieved by any LES. Since the mean velocity is given by the $k_x = k_z = 0$ Fourier-mode, the mean profile is unaffected by filtering in x-z planes. Also, since the mean profile varies very little over a single gridlength, filtering in the y direction does not have any observable effect on the mean velocity. This, however, is not the case for second-order statistics of velocity and vorticity and there explicit filtering must be applied to the DNS data for comparison with the LES. The LES with DM took about 11 minutes of CPU time for the entire simulation to be completed. For the DLM the CPU time depended on the level of convergence required for the solution of the integral equation. We measured the degree of convergence by the rms error in satisfying the integral equation normalized by the maximum value of $\langle C \rangle$, where $\langle \rangle$ denote averaging over xz planes. When it was required that the error as defined above should not exceed 10^{-4} , the DLM used about 18 minutes of CPU time. To test if this level of convergence was adequate, the simulation was rerun with the convergence criterion set at 10^{-9} . There were no observable differences in any of the computed statistics. For comparison, the high resolution DNS of the same flow over the same physical time interval by Moser and Rogers⁴ required about 200 CPU hours. All computations were performed on a CRAY C90.

The gross features of the wake are characterized by the maximum wake deficit δU_m of the mean velocity profile and the wake half-width *b*. The half-width is defined here as the distance between the two points at which the mean velocity deficit is half its maximum value. Figure 1 shows b^2 plotted as a function of the dimensionless time $\tau = t(\delta U_0)^2/\mu$ for the LES using both the DM and DLM models, the filtered DNS, and the LES with the subgrid model turned off. The width grows as $b \sim \sqrt{t}$ in the self-similar region ($\tau \approx 50-100$) as expected. Figure 2 shows the product



FIG. 2. The product of the wake width and the maximum velocity deficit as a function of time using DLM —; DM---; No-model \cdots ; filtered DNS \bullet .

 $b(\delta U_m)$ as a function of τ . All curves exhibit plateaus during the self-similar periods. Note that the Reynolds number $\operatorname{Re}_{b} = b \, \delta U_{m} / \nu$ is constant and just under 2000 in the selfsimilar period because $\mu/\nu = 2000$. The results of all the LES computations agree reasonably well with the filtered DNS. A somewhat surprising result is that even the LES with zero eddy viscosity gives a reasonable prediction for the spreading rate despite the simulation being grossly underresolved. Flow visualization of the instantaneous flow field and plots of energy spectra show large accumulations of small-scale fluctuations at the smallest resolved scales for this "nomodel" case, as is expected in an underresolved simulation. However, even this gross error does not affect the growth rate much except to make it more "wiggly." This is in sharp contrast to past experience in isotropic turbulence. In that flow, the absence of an eddy-viscosity would prevent energy decay of free turbulence and make a steady state impossible in forced turbulence, rendering comparisons with experiments impossible.

Figure 3 shows the mean velocity profile plotted in self-



FIG. 3. The mean wake velocity deficit in self-similar coordinates using DLM —; DM---; No-model \cdots ; filtered DNS \bullet .

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FIG. 4. Normalized velocity statistics in self-similar coordinates using DLM —; DM---; No-model · · · ·; filtered DNS •. The figures are (a) $\langle u^2 \rangle$, (b) $\langle v^2 \rangle$, (c) $\langle w^2 \rangle$, and (d) $\langle uv \rangle$, and all curves shown are from the "self-similar period" of each computation.

similar coordinates $\delta U_* = \delta U/\delta U_m$ and $y_* = y/b$ for $\tau \approx 50-100$. In all cases very good self-similar collapse is observed (even with the subgrid model turned off). Thus, like the growth rate, the mean velocity profile is quite insensitive to the subgrid model.

The second-order velocity statistics $\langle u^2 \rangle$, $\langle v^2 \rangle$, $\langle w^2 \rangle$, and $\langle uv \rangle$ normalized by $(\delta U_m)^2$ are shown for several times during the "self-similar period" in Fig. 4. Here u, v, and w are the velocities in the x, y, and z directions, respectively, with the mean velocity subtracted out. The angular brackets denote averaging over x-z planes. In all cases it is observed that both the DM and the DLM predict the filtered secondorder statistics well. Except for the $\langle uv \rangle$ profile, the quality of the predictions deteriorates if the model is turned off. The better agreement for the $\langle uv \rangle$ profile is expected since it is directly linked to the mean velocity profile $\delta U(y)$ through the x-component of the momentum equation and it has already been seen that $\delta U(y)$ is insensitive to the subgrid model.

The second-order vorticity statistics $\langle \omega_x^2 \rangle$, $\langle \omega_y^2 \rangle$, $\langle \omega_z^2 \rangle$, and $\langle \omega_x \omega_y \rangle$ normalized by $\operatorname{Re}_b \delta U_m^2$ are shown in Fig. 5, also at times during the "self-similar period." Here ω_x , ω_y , and ω_z are the vorticities in the *x*, *y*, and *z* directions, respectively, with the mean vorticity subtracted out. As before, the angular brackets denote averaging over x-z planes. The agreement of both the DM and the DLM predictions with the filtered DNS is seen to be good. The quality of the predictions is significantly degraded when the model is turned off, in which case the magnitudes of the enstrophy components are about four or five times the corresponding filtered DNS levels. Vorticity statistics are a sensitive measure of the scales close to the threshold of the resolution of the LES. The fact that even vorticity statistics are captured by the LES suggests that all of the resolved scales, and not just the lowest wave number modes, are faithfully represented in the simulation. Thus, we use vorticity statistics as a "quality indicator" of the LES rather than as a quantity of practical importance (note that much of the vorticity resides at subgrid scales and the levels found in the LES or filtered DNS are much less than those observed in the DNS).

In Figs. 4 and 5 it is apparent that the self-similar collapse is not perfect but that there is a systematic variation between the curves at different times in the simulation, even when scaled in self-similar variables. This is the case not only for the LES, but also for the filtered DNS. This is an artifact of the filtering procedure itself and can be understood in the following way. The flow evolves self-similarly at constant Reynolds number $\operatorname{Re}_b = b(\delta U_m)/\nu$ (see Fig. 2) in the



FIG. 5. Vorticity statistics in self-similar coordinates using DLM —; DM---; No-model · · · ·; filtered DNS •. The figures are (a) $\langle \omega_x^2 \rangle$, (b) $\langle \omega_y^2 \rangle$, (c) $\langle \omega_z^2 \rangle$, and (d) $\langle \omega_x \omega_y \rangle$, and all curves shown are from the "self-similar period" of each computation.

self-similar region but the length scales increase in time. Thus, as the flow evolves, the energy spectrum shifts to the left. Since the grid size is held fixed, this implies that more and more of the energy becomes "resolved" as the spectrum shifts to lower wave numbers past $k = 2\pi/\Delta$. Therefore the resolved part of the second-order statistics increases with time. This is precisely what is observed in the LES and filtered DNS data and is responsible for the systematic increasing trend during the self-similar period. The problem here could be remedied by adding the subgrid part of the stress, that is by plotting $\langle uv \rangle + \tau_{12}$ instead of $\langle uv \rangle$. This cannot be done, however, for the turbulent intensities $\langle u^2 \rangle$, $\langle v^2 \rangle$, and $\langle w^2 \rangle$ because the diagonal components of the subgrid stress τ_{11} , τ_{22} , and τ_{33} are absorbed into the pressure and not modeled in the present LES. They could be obtained in a more elaborate model such as the DLM with the k-equation⁶ but in this study we have used the simpler version of the DLM that models only the deviatoric part of the stress.

In addition to obtaining quantitative predictions, one also hopes to gain some qualitative understanding of the largescale flow structures from an LES. Thus, it is of interest to see if the model is able to generate structures that look realistic. As an example, typical contour plots of the *u*-velocity at a time during the self-similar period are presented in Fig. 6 over an x-y plane. It is seen that Fig. 6(c) and Fig. 6(d) (LES with model) bear an overall resemblance to Fig. 6(b) (filtered DNS) in the sense that they have a similar number of "eddies" of approximately similar size and shape. However, Fig. 6(e) (LES without model) looks qualitatively different from Fig. 6(b) in that it has a profusion of poorly resolved small-scale structures. A similar statement can be made about the other flow variables.

In summary, mean normalized velocity profiles plotted in self-similar coordinates are insensitive to the choice of subgrid models. The prediction of the self-similar growth of the wake width is improved by the subgrid model, but the results with no model are nevertheless good. Second-order velocity and vorticity statistics are predicted well by both the DM and DLM, but the predictions of these statistics without a model are poor. The flow structures in the LES have a strong visual resemblance to those of the corresponding filtered DNS, but this is not the case if the LES is performed with no subgrid model. The LES results in a very significant savings in CPU time over the corresponding DNS. The results presented here suggest that LES can provide accurate predictions for turbulent free shear flows when information related to small-scale structures is not required.

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FIG. 6. Contour plots of streamwise velocity fluctuation u for (a) DNS, (b) filtered DNS, (c) DLM LES, (d) DM LES, and (e) no-model LES at τ =71.7. Tick marks are at $2\mu/\delta U_0$.

V. AN ALTERNATE SELF-SIMILAR STATE RESULTING FROM FORCING

In this section we investigate the time evolution of a "forced" wake that has the same mass flux deficit as the "unforced" wake in the previous section. Using LES, we first attempt to reproduce the DNS results for the "strongly forced" wake of Moser and Rogers⁴ and Moser, Rogers, and Ewing.⁵ Next we attempt to extend the DNS results in time with a new LES simulation in a larger computational domain in an effort to observe a definitive self-similar regime. In the DNS no sustained self-similar period was achieved for the strongly forced wake, although there was a fairly brief period of approximate self-similarity that was used to generate time-averaged similarity profiles.

The initial conditions used in the DNS of the "forced" wake were generated from the same two turbulent boundary layer realizations used for the unforced flow, but with additional two-dimensional disturbance energy added to them. In order to maintain a $t^{1/2}$ spreading rate, all the two-dimensional Fourier modes in the computation were amplified instead of just a few particular wavelengths. For the



FIG. 7. The square of the wake width as a function of time for the forced wake: \bullet DNS, --- DLM LES, \cdots no-model LES, — DLM LES of the forced wake extended to the larger domain (upper curve), and — DLM LES of the unforced wake (lower curve).

forced case considered here, both the streamwise and crossstream velocity fluctuations (u' and v') of the twodimensional Fourier modes were multiplied by a factor of 20 at $\tau=0$. This amplification is large, increasing the initial disturbance energy by an order of magnitude. The initial conditions for the corresponding LES computation were generated by filtering the evolved DNS field at $\tau=9.56$. This field was chosen rather than that at $\tau=0.0$ to allow the initial cusped mean profile to smooth out somewhat before trying to resolve it on the LES grid. The subgrid model used is DLM. Although this is computationally more expensive than the DM, it is preferred because of its wider applicability.

A. Reproducing the DNS results

The square of the wake width as a function of time in the DLM LES is shown by the dashed line in Fig. 7. The size of the LES is $N_x = 64$, $N_y = 64$ or 96, $N_z = 16$, and Y_0 ranges from 14.0 initially to 64.0 at the end of the simulation, where N_x , N_y , and N_z are the number of modes in the x, y, and z directions, respectively, and Y_0 is the half-width of the y-domain. (The DNS uses up to $N_x = 600$, $N_y = 260$, and $N_z = 160.$) The LES results agree well with the DNS up to the point where both computations become constrained by the computational domain size. After $\tau \approx 65$ both the DNS and the LES computed in the same domain size exhibit decreasing wake widths instead of reaching the expected sustained self-similar growth. The LES is also no longer as good at predicting the wake width after this point, with the LES width decreasing somewhat more rapidly than that of the DNS.

As with the unforced case, contours of u velocity show good agreement between the LES [Fig. 8(c)] and the filtered DNS [Fig. 8(b)]. When no subgrid scale model is used the computation has an excessive level of small-scale structure owing to inadequate energy dissipation [Fig. 8(d)]. Note that at the time shown (τ =26.3), the sample of large-scale eddies in the computational domain is becoming somewhat limited. As the wake continues to spread it is thus quite probable that the computational domain size will become



FIG. 8. Contour plots of streamwise velocity fluctuation u for (a) DNS, (b) filtered DNS, (c) DLM LES, and (d) no-model LES for the forced wake in the smaller domain at τ =26.3. Tick marks are at $2\mu/\delta U_0$, solid contours are positive, dotted contours are negative, and the contour level is $0.10\delta U_0$.

inadequate and the simulation will no longer be a good representation of an unbounded turbulent flow.

In order to demonstrate that the turbulence in these two computations is constrained by the computational domain size (rather than simply being in a transient non-self-similar state), another LES computation in a domain with twice the streamwise extent and four times the spanwise extent of the computations described above was performed. It should be noted that performing a DNS in this expanded domain size is unfeasible with the computational capabilities currently available.

B. Extending the DNS results to a larger flow domain

Generating the initial conditions for the expandeddomain computation requires some care. Ideally we would like to be simulating the same flow, but simply replicating the periodic flow field used previously will not change the flow evolution at all since the initial periodic symmetry will be maintained by the Navier–Stokes equations. It is thus necessary to break this symmetry in the periodically extended initial conditions. Considering the streamwise direction, if the initial computational wave numbers are scaled to be integers, then the periodic doubling of the streamwise direction will introduce new wave numbers that are odd multiples of one half; these wave numbers have zero energy. To break the initial periodic symmetry these Fourier modes must be seeded with some energy. While adding energy to any single one of the wave numbers would break the symmetry, the time required for nonlinear interactions to transfer energy to the rest of them would be long and the size of the large-scale motions would approach the size of the computational domain before the spectra smoothed out. In order to minimize this transient development time we have chosen to initialize energy in nearly all of the newly generated wave numbers. This has been accomplished by simply taking the disturbance energy in wave number k_x and putting the same energy (randomizing the phase of the disturbance) in the new wave number $k_x + \frac{1}{2}$. The only exception is the $k_x = \frac{1}{2}$ wave number, which does not initially receive any energy from the $k_x = 0$ mode. After this, the entire field is rescaled to have the same disturbance energy density as the original computation. The same procedure is used in the spanwise direction although since the domain is four times as large, the content of k_z is propagated to $k_z + \frac{1}{4}$, $k_z + \frac{1}{2}$, and $k_z + \frac{3}{4}$ (again with randomized phase for each new mode).

The initial flow field generated by the above procedure is thus unphysical, unlike the initial field used for the original computations, which were generated from DNS of a turbulent boundary layer. However, it is hoped that this flow will be similar to the previous computation since the initial energy spectra, mean profiles, relative importance of twodimensional disturbances, and other features are the same between the two flows (the mean velocity profiles are identical). The correspondence between the results from this new simulation and the previous forced simulations up to $\tau \approx 35$ provides some evidence that this is indeed the case (see, for example, Fig. 7).

As with the original forced LES calculation, the initial field for the large-domain LES computation was generated (using the above procedure) from the DNS field at τ =9.56. The large-domain LES was begun with N_x =128, N_y =64, N_z =64, and Y_0 =14. The y-domain is periodically remeshed, increasing both Y_0 and N_y in such a way as to keep the y-resolution approximately constant. The choice of Y_0 at each remesh is such that it is large enough to contain all the vorticity in the flow but not so large that CPU time is wasted computing regions of little activity.

The evolution of the square of the wake width for the LES in the extended domain is shown in Fig. 7. It shows significant deviation from the previous computation beyond $\tau \approx 35$ and achieves a sustained period of linear growth, unlike the forced case in the small domain. It is interesting to note that the growth rate during this apparently self-similar period in the large-domain case is about the same as that during the brief approximately self-similar period of the original LES and DNS computations, although the "virtual origins" of the flows are different (i.e. the width curves are parallel but shifted vertically relative to each other in Fig. 7). This difference in virtual origins of the flows is presumably a consequence of the initialization procedure. Because the large-domain computation does achieve a sustained linear



FIG. 9. The product of the wake width and the maximum velocity deficit as a function of time: \bullet DNS, --- DLM LES, · · · · no-model LES, — DLM LES of the forced wake extended to the larger domain (long curve), and — LES of the unforced wake using DLM (short curve).

growth period of the squared wake width, it is likely that the previous computations were indeed constrained by the size of the computational domain by $\tau \approx 65$ if not sooner.

The product of the wake width and centerline velocity deficit for the large-domain LES is plotted along with the previous LES and DNS results in Fig. 9. The product is quite constant beyond $\tau \approx 50$, consistent with self-similar evolution. In the original LES and the DNS the product is not as constant.

It should be noted that although the large-domain forced LES and the unforced LES both exhibit sustained periods of apparent self-similar evolution, these self-similar periods are characterized by markedly different growth rates. The computations are thus indeed supportive of the idea that alternative initial condition-dependent, self-similar states are possible as suggested by the analysis of George¹ and the experiments of Wygnanski *et al.*² and Marasli *et al.*³ Two-dimensional forcing is seen to result in a sustained significant increase in wake growth rate. The linear region in Fig. 7 is well approximated by

$$\frac{b^2(\delta U_0)^2}{\mu^2} = \alpha \frac{(\delta U_0)^2}{\mu} (t - t_0), \tag{11}$$

where $\alpha = 0.26$, $t_0 = 3.1$ in the unforced case but $\alpha = 1.02$ and $t_0 = -5.9$ in the forced case. The dimensionless growth rate β used in Moser, Rogers, and Ewing⁵ can be calculated from α by

$$\beta = \frac{\alpha}{2} \frac{\mu}{b(\delta U_m)},\tag{12}$$

resulting in $\beta = 0.13$ and $\beta = 0.54$ for the unforced and forced cases, respectively. Note that the value of 0.54 for the forced case is close to the value of 0.58 calculated from the approximate self-similar period in the DNS of Ref. 5, and significantly larger than the value of 0.21 quoted for the "weakly forced" case of that work.

The mean velocity profiles plotted in self-similar coordinates for both the large-domain forced LES and the unforced LES are plotted in Fig. 10. For the forced flow the profiles are obtained from a sequence of approximately equispaced



FIG. 10. The mean wake velocity deficit in self-similar coordinates for — the forced (large-domain) LES at various times during the period $73 < \tau < 300$ and for \bullet the unforced flow at $\tau = 62$, 72, and 86.

times during the period $73 < \tau < 300$. The circles for the unforced case are taken from three times during the self-similar period. It is seen that the velocity profiles at different times for the forced case collapse onto a single curve as they do for the unforced flow examined previously. Furthermore, even though the forced wake has a significantly different growth rate, the mean velocity profile (when plotted in self-similar coordinates) has the same form as the corresponding profile from the unforced case. There is, however, a small lateral shift in the forced mean profile relative to the unforced case that results from a shift present in the initial conditions (and maintained by the Navier-Stokes equations). This comes about as a result of the initialization procedure used, which involves large amplification of particular modes and their propagation to nearby uninitialized Fourier modes. This universality of the mean velocity profile shape is consistent with the experiments^{2,3} and the arguments by George,¹ as discussed in Sec. I.

The linear growth of the squared wake width in Fig. 7 appears to begin at $\tau \approx 75$ and continue until $\tau \approx 220$, after which the growth rate appears to increase further. The collapse of the scaled mean velocity profiles is good throughout this period and it thus appears that the flow may be evolving self-similarly during this period. Reynolds stress profiles at times varying from t=73 to $\tau=300$ in the large-domain forced LES computation are shown in Fig. 11. The different curves correspond to the same times used in Fig. 10. Although the collapse of the curves is greatly improved by using the self-similar scalings, the $\langle u^2 \rangle$, $\langle v^2 \rangle$, and lower half of the $\langle uv \rangle$ profiles show a systematic decrease in magnitude until about $\tau \approx 220$, when the growth rate appears to be changing. This suggests that the flow is not yet completely self-similar. Presumably it takes a while for the high levels of $\langle u^2 \rangle$ and $\langle v^2 \rangle$ present in the initial conditions to come into complete equilibrium with the rest of the flow. For the crossstream resolution used in this forced flow, the subgrid contribution to $\langle uv \rangle$ is negligible and the collapse of the resolved component of $\langle uv \rangle$ is good. In the unforced simulation described in Sec. IV the subgrid contribution to $\langle uv \rangle$ decreased from 17% of the resolved amount at $\tau = 24.9$ to 3.5% at $\tau = 125.0$.

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FIG. 11. Reynolds stress profiles in self-similar coordinates for the LES of the forced wake in the large domain during the period $73 < \tau < 300$ (curves generally decreasing in time). The figures are (a) $\langle u^2 \rangle$, (b) $\langle v^2 \rangle$, (c) $\langle w^2 \rangle$, and (d) $\langle uv \rangle$.

The scaled Reynolds stress profiles are not identical in the forced and unforced cases as is evident by comparing Fig. 11 and Fig. 4. First, there is a large difference in magnitude, with the levels in the forced case being up to an order of magnitude larger than those in the unforced flow. Second, the form of the curves is different. The discrepancy cannot be removed by a simple scaling factor, as can be seen from the fact that the $\langle u^2 \rangle$ and $\langle w^2 \rangle$ profiles are not "doublepeaked" in the forced case. This is in agreement with the arguments of George¹ and contrary to what is expected in the classical theory.¹⁴ It is also in agreement with the experimental results of Refs. 2 and 3.

Since the Reynolds shear stress profile is related to the mean velocity profile, it can be shown,^{4,5} that the $\langle uv \rangle$ profile should be identical for all (inviscid) wakes when scaled with the quantity $(\alpha \delta U_m^2)(\mu/b \delta U_m)$ instead of with $(\delta U_m)^2$. In Fig. 12 the Reynolds shear stress $\langle uv \rangle$ has been plotted with this new scaling. The collapse of the profiles for the two flows is quite good once the high levels of $\langle uv \rangle$ on the lower side of the layer stop decreasing at $\tau \approx 160$. The curves for the unforced case increase in time, partly owing to the decreasing fraction of $\langle uv \rangle$ that is associated with the "sub-grid" scales. By comparing Fig. 11 and Fig. 4 it is seen that the ratio $\langle uv \rangle / \delta U_m^2$ is about four times larger in the forced wake than in the unforced flow, so including the layer growth rate in the scaling, as in Fig. 12, does remarkably well in bringing the profiles into agreement.

VI. DISCUSSION

Large-eddy simulations of temporally evolving wakes were performed using the "dynamic localization" subgrid model. Comparison of growth rates, profiles of first- and second-order statistics, and flow structures show good agreement with the DNS results. This, together with previous tests of the subgrid model on other flows, gives us confidence in the method as an accurate and efficient tool for simulations of unbounded turbulent flows.

LES was performed for both unforced and forced wakes and the hypothesis of universal self-similarity was examined in the light of the data from the simulations. It was found that although flow statistics from each simulated wake exhibited self-similar behavior, the wake spreading rates depended on the initial conditions. This is in contrast to the classical picture,¹⁴ which assumes that all wakes with the same momentum deficit asymptotically approach the same self-



FIG. 12. Normalized Reynolds shear stress in self-similar coordinates for the LES using DLM; — forced wake, \bullet unforced wake.

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similar state. The results of this investigation support the theoretical arguments of George¹ for the existence of "multiple self-similar states" and the experimental results of Wygnanski *et al.*,² Marasli *et al.*,³ and others. The numerical and experimental work complement each other well since the inherent strengths and weaknesses of the two approaches are different. An agreement between the two approaches provides a strong indication that the observed effect is indeed real.

Strictly speaking asymptotic results are exact only after the flow has evolved for an infinitely long time. Numerical simulations can only be run for a limited time period and this makes statements about asymptotic states based on simulation results somewhat tentative; we can really only speculate on the plausibility of different proposed asymptotic states. There is no guarantee that if the flow evolved long enough it would not reach the "classical self-similar state." However, even if this were the case, the present results show that there exists at least a significant intermediate period during which there is self-similar evolution with growth rates depending on initial conditions, as predicted by the analysis of George.¹

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